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ON THE ANGLE BETWEEN TWO LINES GIVEN BY THEIR EQUATIONS.

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WORKS on analytic geometry give the following expressions for the angle, φ , between two right lines whose equations are

$$\begin{aligned} A x + B y + C &= 0, \\ A_1 x + B_1 y + C_1 &= 0. \end{aligned}$$

$$\sin \varphi = \frac{(A_1 B - A B_1) \sin \omega}{\sqrt{(A^2 + B^2 - 2AB \cos \omega)} \sqrt{(A_1^2 + B_1^2 - 2A_1 B_1 \cos \omega)}},$$

$$\cos \varphi = \frac{A A_1 + B B_1 - (A B_1 + A_1 B) \cos \omega}{\sqrt{(A^2 + B^2 - 2AB \cos \omega)} \sqrt{(A_1^2 + B_1^2 - 2A_1 B_1 \cos \omega)}};$$

$$\therefore \tan \varphi = \frac{(A_1 B - A B_1) \sin \omega}{A A_1 + B B_1 - (A B_1 + A_1 B) \cos \omega}.$$

Therefore, say these authors, the lines are parallel if $(A_1 B - A B_1) \sin \omega = 0$, since the tangent then becomes zero. Likewise the lines are perpendicular if $A A_1 + B B_1 - (A B_1 + A_1 B) \cos \omega = 0$, because the tangent then becomes infinite.

This reasoning is not conclusive, since it is not proved that these expressions may not both be zero for the same values of A, A_1, B, B_1 , and thus the tangent be *indeterminate*.

I think a better proof of the above proposition might be given as follows.

The equation

$$\begin{aligned} [(A_1 B - A B_1) \sin \omega]^2 + [A A_1 + B B_1 - (A B_1 + A_1 B) \cos \omega]^2 \\ = (A^2 + B^2 - 2AB \cos \omega)(A_1^2 + B_1^2 - 2A_1 B_1 \cos \omega) \end{aligned} \quad (1)$$

is identical.

If the lines are parallel, $\cos \varphi = \cos 0^\circ = 1$. \therefore

$$\begin{aligned} A A_1 + B B_1 - (A B_1 + A_1 B) \cos \omega &= \sqrt{(A^2 - B^2 - 2AB \cos \omega)} \\ &\times \sqrt{(A_1^2 + B_1^2 - 2A_1 B_1 \cos \omega)}. \end{aligned} \quad (2)$$

Squaring (2), subtracting from (1), extracting the square root and dividing by $\sin \omega$, we obtain for the condition of parallelism

$$A_1 B - A B_1 = 0.$$

If the lines are perpendicular, $\sin \varphi = \sin 90^\circ = 1$; \therefore

$$\begin{aligned} (A_1 B - A B_1) \sin \omega &= \sqrt{(A^2 + B^2 - 2AB \cos \omega)} \sqrt{(A_1^2 + B_1^2 - 2A_1 B_1 \cos \omega)}. \\ &\dots \end{aligned} \quad (3)$$

Squaring (3), subtracting from (1) and extracting the square root we get for the condition of perpendicularity

$$A A_1 + B B_1 - (A B_1 + A_1 B) \cos \omega = 0.$$